

**Instructions:** Complete each of the following exercises for practice.

1. Evaluate the line integral  $\int_{\partial R} P \, dx + Q \, dy$  via Green's Theorem.

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| <p>(a) <math>P(x, y) = ye^x</math>, <math>Q(x, y) = 2e^x</math>;<br/> <math>R = [0, 3] \times [0, 4]</math></p>                                                                              | <p>(d) <math>P(x, y) = y^4</math>, <math>Q(x, y) = 2xy^3</math>;<br/> <math>R</math>: the ellipse <math>x^2 + 2y^2 \leq 2</math></p>                     |
| <p>(b) <math>P(x, y) = x^2 + y^2</math>, <math>Q(x, y) = x^2 - y^2</math>;<br/> <math>R</math>: triangle with vertices <math>(0, 1)</math>, <math>(2, 1)</math>, and <math>(1, 0)</math></p> | <p>(e) <math>P(x, y) = y^3</math>, <math>Q(x, y) = -x^3</math>;<br/> <math>R</math>: the ball <math>x^2 + y^2 \leq 4</math></p>                          |
| <p>(c) <math>P(x, y) = y + e^{\sqrt{x}}</math>, <math>Q(x, y) = 2x + \cos(y^2)</math>;<br/> <math>R</math>: the region bounded by <math>y = x^2</math> and <math>x = y^2</math></p>          | <p>(f) <math>P(x, y) = 1 - y^3</math>, <math>Q(x, y) = x^3 + \exp(y^2)</math>;<br/> <math>R</math>: the annulus <math>4 \leq x^2 + y^2 \leq 9</math></p> |

2. Evaluate  $\int_C \mathbf{F} \cdot d\mathbf{r}$  via Green's Theorem.

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| <p>(a) <math>\mathbf{F} = \langle y \cos(x) - xy \sin(x), xy + x \cos(x) \rangle</math><br/> <math>C</math>: the triangle with vertices <math>(0, 0)</math>, <math>(0, 4)</math>, <math>(2, 0)</math></p>                                                                                                                  | <p>(c) <math>\mathbf{F} = \langle y - \cos(y), x \sin(y) \rangle</math><br/> <math>C</math>: the clockwise circle <math>(x - 3)^2 + (y + 4)^2 = 4</math></p>                                  |
| <p>(b) <math>\mathbf{F} = \langle e^{-x} + y^2, e^{-y} + x^2 \rangle</math><br/> <math>C</math>: the arc of <math>y = \cos(x)</math> from <math>(-\frac{\pi}{2}, 0)</math> to <math>(\frac{\pi}{2}, 0)</math><br/> and then the segment connecting <math>(\frac{\pi}{2}, 0)</math> to <math>(-\frac{\pi}{2}, 0)</math></p> | <p>(d) <math>\mathbf{F} = \langle \sqrt{x^2 + 1}, \arctan(x) \rangle</math><br/> <math>C</math>: the triangle with vertices <math>(0, 0)</math>, <math>(1, 1)</math>, <math>(0, 1)</math></p> |